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A Correction to "On Mean Approximation of Holomorphic Functions"

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In Bell [*1*], the following error appears in the Proof of the Theorem (p. 416): The linear functional $l\colon A_{r,p}(V) \to \mathbf{C}$ has the representation

$$l(f) = \iint_V \lambda(w)\, K_V^{r/p}(w,w)\, f(w)\, du\, dv \qquad (f \in A_{r,p}(V)), \tag{1}$$

for some $\lambda \in L_q(V)$, where $1/p + 1/q = 1$. The exponent of $K_V(w,w)$ had been incorrectly given as r. This invalidates the proof, as given, for $1 < p < 2$. However, by making corrections in the formulas of the proof necessitated by (1), one obtains

$$|l(f)| \leqslant \lim\, (c/n) \iint_V |f(w)|\, K_V{}^t(w,w)\, du\, dv, \tag{2}$$

where c is a constant independent of n, and $t = r/p + 1/q$. Therefore, the following result is established instead:

Let $1 \leqslant p < \infty$ and $r < 1 - p/2$. If M and V satisfy the hypotheses of the Theorem, $f \in A_{r,p}(V)$, and

$$\iint_V |f(w)|\, K_V{}^t(w,w)\, du\, dv < \infty \qquad (t = r/p + 1/q), \tag{3}$$

then f is in the closure of M in $A_{r,p}(V)$.

The following result may then be obtained: If the boundary of V is sufficiently regular, then:

(4) $\leqslant$ is an equivalence relation among the components of the complement of the closure of V,

$$\iint_V K_V{}^t(w,w)\, du\, dv < 0, \qquad \text{if } t < \tfrac{1}{2}, \tag{5}$$

and

(6) The bounded holomorphic functions on V are dense in $A_{r,p}(V)$ whenever $r < \frac{1}{2}$ and $1 \leqslant p < \infty$.

If S is a subset of the complement of V, let $M(S)$ denote the linear space of all rational functions, all of whose poles are simple and lie in S. From the above, and from the Proof of the Theorem in Bell [*1*], one sees that in order to conclude that $M(S)$ is dense in $A_{r,p}(V)$ for $1 \leqslant p < 2$ and $r < 1 - p/2$, it would be sufficient to show that for each continuous linear functional $l\colon A_{r,p}(V) \to \mathbf{C}$ which vanishes on $M(S)$, the function

$$h(z) = (-1/\pi) \iint_V \lambda(w)\, K_V^{r/p}(w, w)\, [1/(w - z)]\, du\, dv, \tag{7}$$

vanishes identically outside V. (The function λ in (7) is obtained from any representation of l, as in (1).) Therefore:

If the boundary of V is sufficiently regular, then $M(S)$ is dense in $A_{r,p}(V)$ for $1 \leqslant p < 2$ and $r < 1 - p/2$ if: Given any component X of the complement of the closure of V, there exists a component Y equivalent to X which satisfies at least one of the following conditions:

(i) Y is unbounded,

(ii) $\int_{\partial Y} \log \rho(z, S)\, ds = -\infty$,

(iii) $\sum \rho(z, \partial Y) = \infty \qquad (z \in Y \cap S)$.

Here ∂Y denotes the boundary of Y,

$$\rho(z, F) = \inf\{|z - w| : w \in F\},$$

for any subset F of the complex plane, and ds in (ii) denotes integration with respect to arc length.

Full details will appear in [2].

References

1. D. Bell, On mean approximation of holomorphic functions. *J. Approx. Theory* **1** (1968), 412–419.
2. D. Bell, "On Mean Approximation of Holomorphic Functions by Rational Functions with Simple Poles", Rice University Studies (to appear).